TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2079 Ashwin

Exam.	[redacted]	Back	
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	I/ II	Time	3 hrs.

Subject: - Engineering Mathematics II (*SH 451*)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *The figures in the margin indicate __Full Marks__.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem on homogeneous function of two independent variable. Verify it for $u = x^n \tan^{-1}\left(\frac{y}{x}\right)$. [1+4]
2. Find the minimum value of $x^2 + xy + y^2 + 3z^2$ under the condition $x+2y+4z = 60$. [5]
3. Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dxdy$ by changing the order of integration. [5]
4. Evaluate $\iint xy(x+y)\, dx\, dy$ over the area between $y = x^2$ and $y = x$. [5]
5. Find the distance of the point (1, - 2,3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$. [5]
6. Find the magnitude and equation of the line of shortest distance between $\frac{x-3}{1} = \frac{y-5}{2} = \frac{z-7}{-3}$ and $\frac{x+1}{3} = \frac{y+2}{-4} = \frac{z+3}{1}$ [5]
7. Find the equation of the sphere through the circle $x^2+y^2+z^2=1, 2x+4y+5z=6$ and touching the plane $z=0$. [5]
8. Obtain the equation of the right circular cylinder of radius 4 and axis the lines $x = 2y = -z$. [5]
9. Solve the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ by power series method. [5]
10. Express in terms of Legendre's Polynomials $f(x) = x^3 - 5x^2 + 6x + 1$. [5]
11. Prove the Bessel's function. [5]
$$J_{-\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right\}.$$
12. Show that $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}) = [\vec{a}\,\vec{b}\,\vec{c}]\,\vec{c}$ and deduce that $[\vec{b}\times\vec{c}\;\; \vec{c}\times\vec{a}\;\; \vec{a}\times\vec{b}] = [\vec{a}\,\vec{b}\,\vec{c}]^2$ [5]
13. Prove that necessary and sufficient condition for a vector function $\vec{f}$ of scalar variable t to have constant direction is $\vec{f}\times\frac{d\vec{f}}{dt} = 0$. [5]
14. Find angle between the normal to the surface at $xy = z^2$ at the points (1,4,2) and (-3,-3,3). [5]
15. Test the convergence or divergence of the following series:
$$\frac{1}{2} + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \frac{4^4}{5^5} + \ldots\ldots\ldots\ldots$$
16. Find the interval of convergence, radius of convergence and centre of convergence for $\sum_{n=1}^{\infty} \frac{2^n(x-3)^n}{n+3}$ [5]

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2079 Jestha

Exam.		Back	
Level	BE	Full Marks	80
Programme	All except BAR	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics (*SH 451*)

- Candidates are required to give their answers in their own words as far as practicable.
- *Attempt All questions.*
- *All questions carry equal marks.*
- *Assume suitable data if necessary.*

1. If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{x}{y}$, *show that* $x\frac{\partial u}{\partial y} + y\frac{\partial u}{\partial y} = 0$.
2. Find the extreme value of $x_2+y_2+z_2$ connected by the relations $x+z=1$ and $2y+z=2$.
3. Evaluate $\iint_R xy\,dx\,dy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the first quadrant.
4. Evaluate $\int_0^a\int_x^a \frac{x}{x^2+y^2}\,dx\,dy$ by changing the order of integration.
5. Find the equation of the plane through the points (1,0,-1) and (3,2,2) and parallel to the line $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.
6. Find the magnitude of the line of the shortest distance between the lines $\frac{x}{\ldots} = \frac{y+1}{3} = \frac{z-2}{2}$; $5x - 2y - 3z + 6 = 0,\ x - 3y + 2z - 3 = 0$.
7. Find the radius and center of the circle $x^2+y^2+z^2+x+y+z=4, \quad x+y+z=0.$
8. Define right circular cylinder. Find the equation of the right circular cylinder whose guiding curve is the circle $x^2+y^2+z^2-x-y-z=0, \quad x+y+z=1.$
9. Solve the differential equation $(1-x^2)\,y'' = y$ by power series method.
10. Prove the Legendre's function $x^4 = \frac{1}{35}\left[8P_4(x)+20P_2(x)+7P_0(x)\right]$.
11. Define Bessel function and show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$.
12. Find the volume of the parallelopiped whose edge are $\ldots\vec{j}-\vec{k},\ \vec{i}-\vec{j}+\vec{k},\ \vec{i}+\vec{j}+\vec{k}$.
13. A particle moves along the curve $x = a\cos t$, $y = a\sin t$, $z = bt$. Find its velocity and acceleration at $t = 0$ and $t = \frac{\pi}{2}$.
14. Find the constants a,b,c so that the vector $\vec{v} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (\ldots x+cy+2z)\vec{k}$ is irrotational.

15. Test the convergence of the series

$$2x+\frac{3x^2}{8}+\frac{4x^3}{27}+----+\frac{(n+1)}{n^3}x^4+---$$

16. Find the radius of convergence of the infinite series $\sum_{n=1}^{\infty}\frac{n^n x^n}{n!}$. Also find centre of the interval of convergence.

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TRIBHUVAN UNIVERSITY

[illegible]TITUTE OF ENGINEERING

[illegible]ination Control Division

2078 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH 451)*

- [illegible]ates are required to give their answers in their own words as far as practicable.
- *[illegible] All questions.*
- *[illegible]res in the margin indicate Full Marks.*
- *[illegible]e suitable data if necessary.*

[illegible] Euler's Theorem for Homogeneous function of two variables. If $u = \sec^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then [illegible] $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$ [1+4]

[illegible] the criteria for a function f(x,y) to have local maximum and minimum value? Using the [illegible], evaluate the extreme value of $x^2+y^2+z^2$ subject to the condition $x+y+z=1$. [5]

[illegible] advantage do we get by changing Cartesian integrals into polar form? Change the following [illegible] into Polar and hence evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dydx$ [1+4]

[illegible] triple integration, the volume of the sphere $x^2+y^2+z^2=a^2$ [5]

[illegible] image of the point (1, 3, 4) in the plane $2x-y+z+3=0$. [5]

[illegible] magnitude and equations of shortest distance between the lines $\frac{[illegible]}{[illegible]} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ [5]

[illegible] cross section of an intersecting sphere and a plane? How many spheres pass through [illegible]ection? [illegible] centre and radius of the circle $x^2+y^2+z^2+x+y+z-4=0,\ x+y+z=0$. [1+4]

[illegible] equation of the right circular cylinder of radius 2 and whose axis is the line $\frac{[illegible]}{[illegible]} = \frac{y-2}{1} = \frac{z-3}{2}$. [5]

[illegible] power series method to solve the differential equation $y'' + xy' + y = 0$. [5]

[illegible]$x^3 - 5x^2 + 6x + 1$ in terms of Legendre's polynomials. [5]

[illegible] $x\,J_n'(x) = n\,J_n(x) - x\,J_{n+1}(x)$ where the symbol Jn denote Bessel's function. [5]

[illegible] set of reciprocal system to the set vectors $2\hat{i}+3\hat{j}-\hat{k}$, $-\hat{i}+2\hat{j}-3\hat{k}$ and [illegible]$-2\hat{k}$ [5]

[illegible]y and sufficient condition for the vector function of a scalar variable t have a [illegible]itude is $\vec{a}\cdot\frac{d\vec{a}}{dt} = 0$. [5]

[illegible] angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point [illegible] [5]

[illegible] for convergence or divergence $[illegible] + \frac{x^3}{3.4} + \ldots, \ (x > 0)$ [5]

[illegible] and interval of convergence of the power series: $\frac{[illegible](x+1)^n}{[illegible]}$. [5]

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2078 Poush

Exam.	Back		
Level	BE	Full Marks	80
Programme	All (Except BAR)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH 451)*

- Candidates are required to give their answers in their own words as far as practicable.
- *Attempt **All** questions.*
- *The figures in the margin indicate **Full Marks**.*
- *Assume suitable data if necessary.*

1. If $z=f(x,y)$ and $x=e^{u}+e^{-v}$, $y=e^{-u}-e^{v}$, prove that $\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$. [5]

2. Write the condition for f(x,y) to be maximum or minimum. Examine and find the maximum and minimum value of: x^3+y^3-3axy [5]

3. Show the region of integration of the following integral: $\int_0^2\int_0^{\sqrt{2x-x^2}}\frac{x}{\sqrt{x^2+y^2}}dydx$. Also evaluate the integral using the polar coordinates. [5]

4. Evaluate $\int_0^a\int_0^x\frac{\cos y\,dydx}{\sqrt{(a-x)(a-y)}}$ by changing order of integration. [5]

OR

The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the plane in A, B, C. Applying Dirichlet's integral to find the volume of tetrahedron OABC.

Find the distance of the point (1, -2, 3) form the plane $x-y+z=5$ measured parallel to the line line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ [5]

Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are coplanar and find the equation of plane in which they lie. [5]

Find the center and radius of the circle: $x^2+y^2+z^2+x+y+z=4,\ x+y+z=0$. [5]

Find the equation of cone whose vertex is origin and guiding curve is $\frac{x^2}{4}=\frac{y^2}{9}=\frac{z^2}{1}$, $x+y+z=1$. [5]

OR

Obtain the equation of right circular cylinder of radius 4 and axis the line $x=2y=-z$.

Apply the power series method to solve the following differential equations: [5]

$$y''-4xy'+(4x^2-2)y=0$$

10. Express $f(x)=x^3-5x^2+x+2$ in terms of Legendre's polynomials.

11. Prove that Bessel's function.

$$J_4(x)=\left(\frac{48}{x^3}-\frac{8}{x}\right)J_1(x)+\left(1-\frac{24}{x^2}\right)J_0(x)$$

12. Give the geometrical interpretation of scalar triple product. Find 'p' such that the vectors $2\bar{i}-\bar{j}+\bar{k}, \bar{i}+2\bar{j}+3\bar{k}$ and $3\bar{i}+p\bar{j}+5\bar{k}$ are coplaner.

13. If $\bar{a}, \bar{b}, \bar{c}$ and $\bar{a}', \bar{b}', \bar{c}'$ are reciprocal system of vectors then show that

$$\bar{a}.\bar{a}'+\bar{b}.\bar{b}'+\bar{c}.\bar{c}'=3$$

14. Find angle between the normal to the surface at $xy=z^2$ at the points (1,4,2) and (-3,-3,3).

15. Test for the convergence of the series:

$$1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+\dots\dots\dots+\frac{x^n}{n^2+1}+\dots..$$

16. Find the radius and interval of convergence of the power series: $\sum_{n=1}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}$

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TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2078 Baishakh

Exam.	Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH 451)*

- Candidates are required to give their answers in their own words as far as practicable.
- *Attempt* ***All*** *questions.*
- ***All*** *questions carry equal marks.*
- *Assume suitable data if necessary.*

State Euler's theorem of the homogeneous function and use it to show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3\tan u$ where $u=\sin^{-1}\frac{x^2y^2}{x+y}$.

Find the minimum value of $x^2+y^2+z^2$ subjected to the condition $x+y+z-1=0$ and $xyz+1=0$.

Evaluate $\int_0^a\int_{\sqrt{ax}}^a \frac{y^2\,dy\,dx}{\sqrt{y^4-a^2x^2}}$ by changing the order of integration.

Find by triple integration, volume of sphere: $x^2+y^2+z^2=a^2$.

Find the equation of the line through the point (-2, 3, 4) and parallel to the planes $2x+3y+4z=5$ and $4x+3y+5z=6$.

Find the magnitude and equation of the S. D between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$.

Obtain the equation of the sphere through the circle: $x^2+y^2+z^2=9$, $x-2y+2z=5$ as a great circle.

Find the equation of cone whose vertical angle is $\frac{\pi}{2}$ with vertex at the origin and its axis along the line $x=-2y=z$.

Solve the differential equation: $(1-x^2)\,y''=y$ by the power series method.

Express the polynomial $f(x)=2x^3+6x^2+5x+4$ in terms of Legendre's polynomials.

Define Bessel function and Prove that: $J_{-\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\cos x$.

[illegible]le find a set of vectors reciprocal to the vectors $2\vec{i}-3\vec{j}+4\vec{k}$, $\vec{i}+2\vec{j}-\vec{k}$ and [illegible]$-2\vec{k}$.

[illegible] moves along the curve $x=a\cos t$, $y=a\sin t$, $z=bt$. Find the velocity and acceleration at $t=0$ and $t=\frac{\pi}{2}$.

[illegible] constant vector and $\vec{r}$ be the position of vector, then prove that [illegible]$)=2\vec{a}$.

[illegible] infinite series $\frac{1^2}{3^1}+\frac{2^2}{3^2}+\frac{3^2}{3^3}+\ldots$ whether it is divergent or convergent.

[illegible] radius and interval of convergence of the power series: $\sum_{n=1}^{\infty}\frac{(-1)^n}{n.2^n}x^n$

TRIBHUVAN UNIVERSITY INSTITUTE OF ENGINEERING **Examination Control Division** 2077 Chaitra	Exam.	Regular		
	Level	BE	Full Marks	80
	Programme	All (Except B.Arch.)	Pass Marks	32
	Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH 451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Give an example of Homogenous function. State Euler's theorem on Homogenous function for two independent variable x and y. Verify Euler's theorem for $u=x^n \sin\left(\frac{y}{x}\right)$.
2. Find the extreme value of $x^2+y^2+z^2$ connected by the relation $x+z=1$ and $y+z=2$
3. Evaluate the double integration $\int_0^1\int_x^{\sqrt{2-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$.
4. Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$.
5. Prove that the lines $x=\frac{y-2}{2}=\frac{z+3}{3}$ and $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ are coplanar and find their plane and point of intersection.
6. Find the equation of shortest distance between the lines $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$.
7. Find the radius and centre of the circle $x^2+y^2+z^2-8x+4y+8z-45=0,\ x-2y+2z-3=0$
8. Find the equation of the cone with vertex (1, 1, 0) and guiding curve is $y=0,\ x^2+z^2=4$.
9. Solve by the power series method of the differential equation $y''-y=0$.
10. Test whether the solutions of $y'''-2y''-y'+2y=0$ are linearly independent or dependent.
11. Show that $4J''_n(x)=J_{n-2}(x)-2J_n(x)+J_{n+2}(x)$
12. Prove that $\vec{i}\times(\vec{a}\times\vec{i})+\vec{j}\times(\vec{a}\times\vec{j})+\vec{k}\times(\vec{a}\times\vec{k})=2\vec{a}$ where $\vec{i},\ \vec{j},\ \vec{k}$ are the mutually unite vectors along the coordinates axes.
13. The necessary and sufficient condition for the function $\vec{a}$ of a scalar variable t to have a constant direction is $\vec{a}\times\frac{d\vec{a}}{dt}=0$.
14. Find the directional derivatives of $\varphi(x,y,z)=xy^2+yz^3$ at the point (2, -1, 1) in the direction of vector $\vec{i}+2\vec{j}+2\vec{k}$.
15. Test the convergence of the series $\frac{2}{3}x+\frac{3}{2^3}x^2+\frac{4}{3^3}x^3+\ldots+\frac{(n+1)^n}{n^3}x^n+\ldots$

$(-1)^n(x-3)^n$

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2076 Baishakh

Exam.	[redacted]	Full Marks	80
Level	BE	Pass Marks	32
Programme	All (Except BAR)	Time	3 hrs.
Year / Part	I / II		

Subject: **- Engineering Mathematics II *(SH 451)***

- Candidates are required to give their answers in their own words as far as practicable.
- *Attempt **All** questions.*
- ***All** questions carry equal marks.*
- *Assume suitable data if necessary.*

1. State and prove Euler's theorem for a homogeneous function of two variables.
2. Find the extreme value of $x^2 + y^2 + z^2$ subject to the conditions: $x + y + z = 1$ and $xyz + 1 = 0$.
3. Evaluate $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2+y^2}$ by changing order of integration.
4. Find the volume of $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ using Drichelet's integral.
5. Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $4x - 3y + 1 = 0,\ 5x - 3z + 2 = 0$ are coplanar. Also find the point of intersection.
6. Find the magnitude and equation of shortest distance between the lines: $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$.
7. Find the radius and centre of the circle: $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0,\ x - 2y + 2z - 3 = 0$
8. Find the equation of cone with vertex (1, 2, 3) and the base $9x^2 + 4y^2 = 36,\ z = 0$.
9. Find the reciprocal system of vector of the set of vectors: $-\vec{i}+\vec{j}+\vec{k},\ \vec{i}-\vec{j}+\vec{k}$ and $\vec{i}+\vec{j}-\vec{k}$.
10. For the curve $x = 3t,\ y = 6t^2,\ z = 4t^3$, prove that $\left[\dot{\vec{r}}\ \ddot{\vec{r}}\ \dddot{\vec{r}}\right] = 864$
11. If $\vec{r} = x\vec{i}+y\vec{j}+z\vec{k}$ and $\vec{a},\vec{b}$ are constant vectors, then prove that $\operatorname{curl}\left(\vec{b}\times\left(\vec{r}\times\vec{a}\right)\right)=\vec{a}\times\vec{b}$.
12. Solve by the power series method the differential equation: $y''-4xy'+(4x^2-2)y=0$.
13. Solve the Legendre's equation: $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$.
14. Prove that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$.
15. Test the convergence of the series: $\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+\ldots.. \quad (x>0)$.
16. Find the interval and radius of convergence of power series:

02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2075 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *__All__ questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem of homogeneous function and use it to show $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-\frac{1}{6}\tan u$ where $u=\operatorname{cosec}^{-1}\left(\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right)$.

2. Find the maximum value of $f(x, y, z) = xyz$ when $x + y + z = 9$.

3. Show the region of integration of the following integral:

$$\int_0^1\int_x^{\sqrt{2-x^2}}\frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$$

Also evaluate the integral using polar coordinates.

4. Evaluate $\iiint_V x\,dxdydz$ where V is the region in the first octant bounded by the surface $x^{2/3}+y^{2/3}+z^{2/3}=a^{2/3}$.

5. Find the distance from the point (3, 4, 5) to the point where the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ meets the plane $x+y+z=2$.

6. Find the magnitude and equation of shortest distance between the lines $\frac{x-3}{1}=\frac{y-5}{2}=\frac{z-7}{-3}$ and $\frac{x+1}{3}=\frac{y+2}{-4}=\frac{z+3}{1}$.

7. Find the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0,\ x+y+z=3$ as a great circle. Also determine its center and radius.

8. Prove that the equation $2x^2+y^2+3z^2+4x+2y+6z+d=0$ represents a cone if d = 6.

9. Define scalar triple product of three vectors. State its geometrical meaning and hence find the volume of the parallelopiped whose concurrent edges are:

$\vec{i}+2\vec{j}-\vec{k},\ \vec{i}-\vec{j}+\vec{k}$ and $\vec{i}+\vec{j}+\vec{k}$.

10. Prove that the necessary and sufficient condition for the vector function $\vec{a}(t)$ of scalar variable t to have constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$.

11. Find the directional derivative of $\phi(x, y, z) = x^2 + yz + 4xz^2$ at the point (1, -2, -1) in the direction of vector $2\hat{i} - \hat{j} - 2\vec{k}$.

12. Apply Power series method to solve the following differential equation:

$$(2-x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 0$$

13. Express the polynomial $f(x) = 2x^3 + 6x^2 + 5x + 4$ in terms of Legendre's polynomials.

14. Show that $J_{-5/2}(x) = \sqrt{\frac{2}{n\pi}}\left[\frac{3}{x}\sin x + \frac{3-x^2}{x^2}\cos x\right]$

15. Test the convergence of the series:

$$\sum_{n=0}^{\infty}\frac{x^n}{n^2+2} \text{ where } x > 0.$$

16. Find the internal and radius of convergence of power series:

$$\sum_{n=1}^{\infty}\frac{(-1)^n(2x+1)^n}{3^n}$$

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2075 Baishakh

Exam.	Back		
Level	BE	Full Marks	80
Programme	ALL (Except B. Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's Theorem for homogeneous function of two variables. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. [1+4]

2. Obtain the maximum value of xyz such that $x+y+z=24$. [5]

3. Evaluate: $\int_0^a\int_{\sqrt{ax}}^{a}\frac{y^2}{\sqrt{y^4-a^2x^2}}dydx$ by changing order of integration. [5]

4. Evaluate: $\iiint_R (2x+y)dx\,dy\,dz$ where R is closed region sounded by cylinder $z=4-x^2$ and planes $x=0,\ y=0,\ y=2,\ z=0$. [5]

5. Show that $\frac{x+4}{3}=\frac{y+6}{5}=\frac{z-1}{-2}$ and $3x-2y+z+5=0=2x+3y+4z-4$ are coplanar lines and find the point of intersection. [5]

6. Show that the shortest distance between the lines $x+a=2y=-12z$ and $x=y+2a=6z-6a$ is 2a. [5]

7. Obtain the equation of tangent plane to sphere $x^2+y^2+z^2+6x-2z+1=0$ which passes through the line $3(16-x)=3z=2y+30$ [5]

8. Find the equation of cone with vertex at $(3,1,2)$ and base $2x^2+3y^2=1,\ z=1$ [5]

   OR

   Find the equation of the right circular cylinder whose guiding curve is the circle: $x^2+y^2+z^2-x-y-z=0,\ x+y+z=1$

9. Solve the initial value problem: $y''-4y'+3y=10e^{-2x},\ y(0)=1,\ y'(0)=3$ [5]

10. Solve the differential equation by power series method: $y''-y=0$ [5]

11. Solve in series, the Legendre's equation $\left(1-x^2\right)y''-2xy'+n(n+1)y=0$ [5]

OR

Prove the Bessel's function $J_{\frac{3}{2}}(x)=\sqrt{\frac{2}{\Pi x}}\left(\frac{\sin x}{x}-\cos x\right)$

12. Prove that $\left[\vec{a}\times\vec{b}\ \ \vec{c}\times\vec{d}\ \ \vec{e}\times\vec{f}\right]=\left[\vec{a}\,\vec{b}\,\vec{d}\right]\left[\vec{c}\,\vec{e}\,\vec{f}\right]-\left[\vec{a}\,\vec{b}\,\vec{c}\right]\left[\vec{d}\,\vec{e}\,\vec{f}\right]$ [5]

13. Prove that the necessary and sufficient conditions for the vector function $\vec{a}$ of scalar variable t to have constant direction is $\vec{a}\times\frac{d\vec{a}}{dt}=0$ [5]

14. Find the angle between the normal to the surfaces given by: $x\log z=y^2-1$ and $x^2y+z=2$ at the point $(1,1,1)$ [5]

15. Test the convergence of the series: [5]

$$x+\frac{3}{5}x^2+\frac{8}{10}x^3+\frac{15}{17}x^4+......+\frac{n^2-1}{n^2+1}x^n+......,x>0.$$

16. Find the interval and radius of convergence of power series: [5]

$$\frac{1}{1.2}(x-2)+\frac{1}{2.3}(x-2)^2+\frac{1}{3.4}(x-2)^3+........+\frac{1}{n(n+1)}(x-2)^n+............$$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2074 Bhadra

Exam.	**Regular**		
Level	BE	**Full Marks**	80
Programme	ALL (Except B. Arch.)	**Pass Marks**	32
Year / Part	I / II	**Time**	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $u = \log\dfrac{x^2+y^2}{x+y}$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 1$
2. Find the minimum value of $x^2+y^2+z^2$ when $x+y+z=3a$.
3. Evaluate $\iint xy(x+y)\,dxdy$ over the area between $y=x^2$ and $y=x$
4. Evaluate $\int_0^a\int_y^a \dfrac{x\,dx\,dy}{x^2+y^2}$ by changing order of integration.

OR

Evaluate $\iiint_v x^2dx\,dy\,dz$ over the region v boundary by the planes x = 0, y =0, z = 0 and x+y+z=a

5. Obtain the equation of the plane passing through the line of intersection of two planes through the line of intersection of two planes $7x-4y+7z+16=0$ and $4x-3y-2z+13=0$ and perpendicular to plane $x-y-2z+5=0$
6. Find the length and equation of the shortest distance between the lines $\dfrac{x-3}{3}=\dfrac{y-8}{-1}=\dfrac{z-3}{1}$, $2x-3y+27=0;\ 2y-z+20=0$
7. Find the equation of the sphere having the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z-8=0$ as a great circle.
8. Find the equation of right circular cone whose vertex at origin and axis is the line $\dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{3}$ with vertical angle 30°

   OR

   Find the equation of the right circular cylinder of radius 2 whose axis is the line $\dfrac{x-1}{2}=\dfrac{y-2}{1}=\dfrac{z-3}{2}$
9. Solve by power series method the differential equation $y''+xy'+y=0$
10. Express the following in terms of legendre's Polynomials $f(x)=5x^3+x$

11. Prove the Bessel's function $J_{-\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3}{x}\sin x + \frac{3-x^2}{x^2}\cos x\right]$

12. Find the set of reciprocal system to the set of vectors $2\vec{i}+3\vec{j}-\vec{k}, -\vec{i}+2\vec{j}-3\vec{k}$ and $3\vec{i}-4\vec{j}+2\vec{k}$

13. Prove that the necessary and sufficient condition for the vector function of scalar variable 't' have constant magnitude is $\vec{a}.\frac{d\vec{a}}{dt}=0$

14. Find the directional derivative of $\phi(x, y, z) = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of vector $\vec{i}+2\vec{j}+2\vec{k}$

OR

If $\vec{a}$ is a constant vector and $\vec{r}$ be the position vector then prove that $\left(\vec{a}\times\nabla\right)\times\vec{r} = -2\vec{a}$

15. Test convergent or divergent of the series $1+\frac{x}{2}=\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+........\infty$

16. Find the internal and radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{2^n(x-3)^n}{n+3}$

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2073 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B. Arch)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *__All__ questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $u = \log\left(\dfrac{x^2+y^2}{x+y}\right)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 1$.

2. Obtain the maximum value of xyz such that x+y+z = 24.

3. Evaluate: $\iint xy(x+y)dxdy$ over the area between $y = x^2$ and $y = x$.

4. Evaluate the integral by changing to polar co-ordinates: $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\, dydx$

OR

Find by triple integration the volume of the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

5. Show that the lines $\dfrac{x-1}{2} = \dfrac{y+1}{-3} = \dfrac{z+10}{8}$ and $\dfrac{x-4}{1} = \dfrac{y+3}{-4} = \dfrac{z+1}{7}$ are coplanar. Find their common point.

6. Find the S.D between the lines $\dfrac{x-6}{3} = \dfrac{y-7}{-1} = \dfrac{z-4}{1}$ and $\dfrac{x}{-3} = \dfrac{y+9}{2} = \dfrac{z-2}{4}$. Find also the equation of shortest distance.

7. Find the equation of spheres passing through the circle $x^2+y^2+z^2-6x-2z+5 = 0$, $y = 0$ and touching the plane $3y+4z+5 = 0$.

8. Find the equation of the cone whose vertex is the origin and base the cirole $y^2+z^2 = b^2$, and $x = a$.

OR

Find the equation to the right circle cylinder of radius 2 and whose is the line $\dfrac{x-1}{2} = \dfrac{y-2}{1} = \dfrac{z-3}{2}$.

9. Solve by Power series method $y'' - y = x$.

10. Express in terms of Legendre's polynomials $f(x) = x^3-5x^2+6x+1$.

11. Prove the Bessel's Function

$$J_{3/2}(x) = \sqrt{\frac{2}{\Pi x}}\left(\frac{\sin x}{x} - \cos x\right)$$

12. Find the set of reciprocal system to the set of vectors $2\vec{i}+3\vec{j}-\vec{k}$, $-\vec{i}+2\vec{j}-3\vec{k}$, and $3\vec{i}-4\vec{j}+2\vec{k}$.

13. Prove that $\left[\vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}\quad \vec{a}\times\vec{b}\right]=\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

14. If $\vec{r}$ be the position vector and $\vec{a}$ is constant vector then prove that $\nabla\left(\frac{\vec{a}\bullet\vec{r}}{r^n}\right)=\frac{\vec{a}}{r^n}-\frac{n}{r^{n+2}}\left(\vec{a}\bullet\vec{r}\right)\vec{r}$

OR

Find the value of n so that $r^n\,\vec{r}$ is solenoidal.

15. Test the series for convergence or divergence

$$2x+\frac{3x^2}{8}+\frac{4x^3}{27}+\dots\dots\dots+\frac{(x+1)x^n}{n^3}+\dots\dots\dots\quad (x>0)$$

16. Find the interval of convergence and the radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{2^n(x-3)^n}{n+3}$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2073 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's Theorem for a homogeneous function of two independent variables and verify it for the function $u = x^n \tan^{-1}\left(\frac{y}{x}\right)$

2. Find the extreme value of $x^2+y^2+z^2$ connected by the relation $ax+by+cz = p$.

3. Evaluate $\iint xy(x+y)dxdy$ over the area between $y = x^2$ and $y = x$.

4. Evaluate the integral by changing to polar coordinates $\int_0^a\int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\,dy.dx$

OR

Find by triple integration the volume of the sphere $x^2+y^2+z^2 = a^2$.

5. Find the equation of the plane through the line $2x+3y-5z = 4$ and $3x-4y+5z = 6$ and parallel to the coordinate axes.

6. Find the length and equation of shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $2x-3y+27 = 0$, $2y-z+20 = 0$.

7. Obtain the centre and radius of the circle $x^2+y^2+z^2+x+y+z = 4$, $x+y+z = 0$.

8. The plane through OX and OY includes an angle α, prove that their line of intersection lies on the cone $z^2(x^2+y^2+z^2) = x^2y^2\tan^2\alpha$

OR

Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2}$.

9. Solve by power series method the differential equation $y'' - 4xy' + (4x^2 - 2)y = 0$,

10. Express $f(x) = x^3 - 5x^2 + x + 2$ in terms of Legendre's polynomial.

11. Show that $J_{-\left(\frac{5}{2}\right)}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{3}{x}\sin x + \frac{3-x^2}{x^2}\cos x\right)$.

12. Prove that $\left[\vec{b}\times\vec{c}\;\;\vec{c}\times\vec{a}\;\;\vec{a}\times\vec{b}\right]=\left[\vec{a}\;\vec{b}\;\vec{c}\right]^2$

13. A partical moves along the curve x = a cost, y = a Sint and z = bt. Find the velocity and acceleration at t = 0 and t = π/2.

14. Find the directional derivative of $\phi(x, y, z) = xy^2+yz^3$ at the point (2, -1, 1) in the direction of vector $\vec{i}+2\vec{j}+2\vec{k}$.

OR

If $\vec{a}$ is a constant vector and $\vec{r}$ be the position vector then prove that $(\vec{a}\times\nabla)\times\vec{r}=-2\vec{a}$.

15. Test the convergence of the series

$$\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+\ldots\ldots\ldots\ldots, x>0$$

16. Find the interval and radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}$.

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2071 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem for a homogeneous function of two independent variables and verify it for the function $u = x^n.\sin\left(\frac{y}{x}\right)$. [1+4]

2. Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 1$ and $xyz + 1 = 0$. [5]

3. Evaluate $\iint xy(x+y)dxdy$ over the area between $y = x^2$ and $y = x$. [5]

4. Evaluate the integral by changing to polar coordinates $\int_0^1\int_x^{\sqrt{2x-x^2}}(x^2+y^2)dy.dx$. [5]

OR

Find by triple integration the volume of sphere $x^2 + y^2 + z^2 = a^2$. [5]

5. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $4x - 3y + 1 = 0 = 5x - 3z + 2$ are coplanar. Also find their point of intersection. [5]

6. Find the length and equation of the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $2x - 3y + 27 = 0, 2y - z + 20 = 0$. [5]

7. Find the centre and radius of the circle $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$, $x - 2y + 2z - 3 = 0$. [5]

8. Find the equation of right circular cone whose vertex at origin and axis the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ with the vertical angle 30°. [5]

OR

Find the equation of the right circular cylinder having for its base the circle $x^2 + y^2 + z^2 = 9, x - y + z = 3$. [5]

9. Solve by the power series method the differential equation $y'' - 4xy' + (4x^2 - 2)y = 0$. [5]

10. Test whether the solutions of $y''' - 2y'' - y' + 2y = 0$ are linearly independent or dependent. [5]

11. Show that: $J_{-\left(\frac{5}{2}\right)}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{3}{x}\sin x + \frac{3-x^2}{x^2}\cos x\right)$ [5]

12. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are the reciprocal system of vectors, then prove that

$\vec{a}'\times\vec{b}' + \vec{b}'\times\vec{c}' + \vec{c}'\times\vec{a}' = \frac{\vec{a}+\vec{b}+\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $[\vec{a}\ \vec{b}\ \vec{c}] \neq 0$. [5]

13. The necessary and sufficient condition for the function $\vec{a}$ of scalar variable t to have a constant direction is $\vec{a}\times\frac{d\vec{a}}{dt} = 0$. [5]

14. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of vector $2\vec{i} - \vec{j} - 2k$. [5]

OR

If $\vec{a}$ is a constant vector and $\vec{r}$ be the position vector, then, prove that $\nabla\times(\vec{a}\times\vec{r}) = 2\vec{a}$. [5]

15. Determine whether the series is convergent or divergent $\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1} - n\right)$ [5]

16. Find the interval and radius of convergence of the power series: $\sum_{n=1}^{\infty}\frac{2^n.(x-3)^n}{n+3}$. [5]

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2070 Bhadra

Exam.	**Regular**		
Level	BE	**Full Marks**	80
Programme	All (Except B.Arch.)	**Pass Marks**	32
Year / Part	I / II	**Time**	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *__All__ questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $u = \log\dfrac{x^2+y^2}{x+y}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 1$.
2. Find the extreme value of $x^2 + y^2 + z^2$ connected by the relation $ax + by + cz = p$.
3. Evaluate $\displaystyle\int_0^a \int_{\sqrt{ax}}^a \frac{y^2dydx}{\sqrt{y^4 - a^2x^2}}$ by changing order of integration.
4. Evaluate $\displaystyle\int_0^{\log 2}\int_0^x\int_0^{x+\log y} e^{x+y+z}dzdydx$.
5. Find the length of the perpendicular from the point (3, −1, 11) to the line $\dfrac{x}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$. Also obtain the equation of perpendicular.
6. Find the magnitude and the equation of S.D. between the lines $\dfrac{x-3}{3} = \dfrac{y-8}{-1} = \dfrac{z-3}{1}$ and $2x - 3y + 27 = 0, 2y - z + 20 = 0$.
7. Find the equation of the sphere through the circle $x^2 + y^2 = 4$, $z = 0$ and is intersected by the plane $x + 2y + 2z = 0$ is a circle of radius 3.

   OR

   Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 + 6x - 2z + 1 = 0$ which passes through the line $x + z - 16 = 0, 2y - 3z + 30 = 0$.
8. Find the equation of the right circular cone whose vertex at origin and axis is the line $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$ with vertical angle 30°.

   OR

   Find the equation of the right circular cylinder of radius 2 whose axis is the line $\dfrac{x-1}{1} = \dfrac{y-2}{1} = \dfrac{z-3}{2}$.
9. Solve the differential equation $y'' - 4xy' + (4x^2 - 2)y = 0$ by power series method.
10. Express $f(x) = x^3 - 5x^2 + x + 2$ interms of Legendre polynomials.

11. Show that $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$.

12. Find a set of vectors reciprocal to the following vectors $2\vec{i}+3\vec{j}-\vec{k}$, $\vec{i}-\vec{j}-2\vec{k}$, $-\vec{i}+2\vec{j}+2\vec{k}$.

13. Prove that the necessary and sufficient condition for the vector function of a scalar variable t to have constant magnitude is $\vec{a}\cdot\frac{d\vec{a}}{dt}=0$.

14. A particle moves along the curve $x = 4\cos t$, $y = t^2$, $z = 2t$. Find velocity and acceleration at time $t = 0$ and $t = \frac{\pi}{2}$.

15. Test the convergence of the series $1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\ldots$

16. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-1)^n(x-3)^n}{n+1}$.

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2070 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *__All__ questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Find $\frac{du}{dt}$ if $u=\sin\left(\frac{x}{y}\right)$, $x=e^t \;\&\; y=t^2$

2. Find the extreme value of $x^2+y^2+z^2$ connected by the relation x+z = 1 and 2y+z = 2

3. Evaluate: $\iint_R xy\,dx.dy$ where R is the region over the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the first quadrant.

4. Evaluate the integral by changing to polar coordinates $\int_0^a\int_0^{\sqrt{a^2-x^2}} y^2.\sqrt{x^2+y^2}.dy.dx$

   OR

   Evaluate: $\iiint x^{l-1}.y^{m-1}.z^{n-1}.dx.dy.dz$, where x,y,z are all positive but $\left(\frac{x}{a}\right)^p+\left(\frac{y}{b}\right)^q+\left(\frac{z}{c}\right)^r\le 1$

5. Find the equation of the plane through the line 2x+3y-5z = 4 and 3x-4y+5z = 6 and parallel to the coordinates axes.

6. Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z-3}{-5}$ & $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are coplanar. Find their point of intersection and equation of plane in which they lie.

7. Find the centre and radius of the circles $x^2+y^2+z^2-8x+4y+8z-45=0$, x-2y+2z-3=0

8. Find the equation of a right circular cone with vertex (1,1,1) and axis is the line $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ and semi vertical angle 30°.

9. Solve by power series method the differential equation $y''+xy'+y=0$

10. Find the general solution of the Legendre's differential equation.

11. Prove Bessel's Function $\frac{d\left[x^{-n}J_n(x)\right]}{dx}=-x^{-n}J_{n+1}$

12. Prove that: $\left[\vec{b}\times\vec{c}\;\;\vec{c}\times\vec{a}\;\;\vec{a}\times\vec{b}\right]=\left[\vec{a}\;\vec{b}\;\vec{c}\right]^2$

10/2

13. Find n so that $r^n \vec{r}$ is solonoidal.

14. Prove that the necessary and sufficient condition for a function $\vec{a}$ of scalar variable to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$

15. Test the series for convergence or divergence

$$x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \frac{15}{17}x^4 + \ldots\ldots + \frac{n^2-1}{n^2+1}x^n + \ldots..(x > 0)$$

16. Find the radius of convergence and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n.2^n}$

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2069 Bhadra

Exam.	**Regular (2066 & Later Batch)**		
Level	BE	**Full Marks**	80
Programme	All	**Pass Marks**	32
Year / Part	I / II	**Time**	3 hrs.

Subject: - Engineering Mathematics II ***(SH451)***

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $\sin u = \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$, show that $x\dfrac{\delta u}{\delta x}+y\dfrac{\delta x}{\delta y}=0$.
2. Obtain the maximum value of xyz such that $x+y+z=24$.
3. Evaluate: $\iint xy(x+y)dxdy$ over the area between $y=x^2$ and $y=x$.
4. Evaluate $\iiint_V x^2\,dx\,dy\,dz$ over the region V bounded by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=a$.
5. Find the image of the point $(2, -1, 3)$ in the plane $3x-2y-z-9=0$.
6. Find the S.D. between the line $\dfrac{x-6}{3}=\dfrac{7-y}{1}=\dfrac{z-4}{1}$ and $\dfrac{x}{-3}=\dfrac{y+9}{2}=\dfrac{2-z}{-4}$. Find also equation of S.D.
7. Obtain the equation of the sphere through the circle $x^2+y^2+z^2=9$, $x-2y+2z=5$ as a great circle.
8. Find the equation of cone with vertex $(3, 1, 2)$ and base $2x^2+3y^2=1$, $z=1$.

   OR

   Find the equation of right circular cylinder whose axis is the line $\dfrac{x-\alpha}{\ell}=\dfrac{y-\beta}{m}=\dfrac{z-r}{n}$ and whose radius 'r'
9. Solve the initial value problem $y''+2y'+5y=0$, given $y(0)=1$, $y'(0)=5$.
10. Define power series. Solve by power series method of differential equation, $y'+2xy=0$.
11. Prove the Bessell's function $\dfrac{d}{dx}\left[x^n J_n(x)\right]=x^n J_{n-1}(x)$.
12. Prove if $\vec{\ell}, \vec{m}, \vec{n}$ be three non-coplanar vectors then

$$\left[\vec{\ell}\,\vec{m}\,\vec{n}\right]\left(\vec{a}\times\vec{b}\right)=\begin{vmatrix}\vec{\ell}.\vec{a} & \vec{\ell}.\vec{b} & \vec{\ell}\\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m}\\ \vec{n}.\vec{a} & \vec{n}.\vec{b}, & \vec{n}\end{vmatrix}.$$

3

13. Prove that the necessary and sufficient condition for the vector function of a scalar variable t have a constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$.

14. Find the angle between the normal to the surfaces $x \log z = y^2 - 1$ and $x^2 y + z = 2$ at the point (1, 1, 1).

15. Test the convergence of the series $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \ldots\ldots\ldots\ldots$

16. Find the interval of cgt, radius of cgt and centre of cgt of power series $\sum \frac{2^n x^n}{n!}$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2069 Poush

Exam.	**New Back (2066 & Later Batch)**		
Level	BE	**Full Marks**	80
Programme	All except B.Arch.	**Pass Marks**	32
Year / Part	I / II	**Time**	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem on homogeneous functions of two independent variables. And if $\text{Sin } u = \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ then prove $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=0$

2. Find the minimum value of the function $F(x,y,z)=x^2+y^2+z^2$ when $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=1$

3. Evaluate: $\iint r^3 dr\, d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$

4. Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dz\, dx\, dy$

OR

Find the volume of sphere $x^2+y^2+z^2 = a^2$ using Diritchlet's integral.

5. Prove that the lines
$\dfrac{x+1}{-3}=\dfrac{y-3}{2}=\dfrac{z+2}{1}$ and $x=\dfrac{y-7}{-3}=\dfrac{z+7}{2}$ are coplanar and find the equation of plane in which they lie.

6. Show that the shortest distance between two skew lines
$\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ and $\dfrac{x-2}{3}=\dfrac{y-4}{4}=\dfrac{z-5}{5}$ is $1/\sqrt{6}$

7. A variable plane is parallel to the plane $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ and meets the axes in A, B, C. Prove that the circle ABC lies on the cone $\left(\dfrac{b}{c}+\dfrac{c}{b}\right)yz+\left(\dfrac{c}{a}+\dfrac{a}{c}\right)zx+\left(\dfrac{a}{b}+\dfrac{b}{a}\right)xy=0$

8. Find the equation of the right circular cylinder of radius 4 and axis the line $x = 2y = -z$

9. Show that the solutions of $x^2y'''-3xy''+3y'=0,(x>0)$ are linearly independent.

OR

Solve the equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-4)y=0$ in series form.

10. Prove that $4J''_n(x)=J_{n-2}(x)-2J_n(x)+J_{n+2}(x)$ where the symbols have their usual meanings.

11. Apply the power series method to the following differential equation $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

OR

Find the general solution of Legendre's differential equation.

12. Show that $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})=\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{c}$ and deduce $\left[\vec{b}\times\vec{c}\;\;\vec{c}\times\vec{a}\;\;\vec{a}\times\vec{b}\right]=\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$

13. Prove that the necessary and sufficient condition for the function $\vec{a}$ of scalar variable to have a constant direction is $\vec{a}\times\frac{d\vec{a}}{dt}=0$

14. Find the angle between the surface $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2,-1,2)

15. Test the convergence of the series $\sum\frac{(n+1)^n x^n}{n^{n+1}}$

16. Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-1)^n(x-1)^2}{\sqrt{n}}$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2069 Poush

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All except B.Arch.	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem on homogeneous functions of two independent variables. And if $\operatorname{Sin} u = \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ then prove $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=0$

2. Find the minimum value of the function $F(x,y,z)=x^2+y^2+z^2$ when $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=1$

3. Evaluate: $\iint r^3 dr\, d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$

4. Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\, dz\, dx\, dy$

OR

Find the volume of sphere $x^2+y^2+z^2 = a^2$ using Diritchlet's integral.

5. Prove that the lines

$\dfrac{x+1}{-3}=\dfrac{y-3}{2}=\dfrac{z+2}{1}$ and $x=\dfrac{y-7}{-3}=\dfrac{z+7}{2}$ are coplanar and find the equation of plane in which they lie.

6. Show that the shortest distance between two skew lines

$\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ and $\dfrac{x-2}{3}=\dfrac{y-4}{4}=\dfrac{z-5}{5}$ is $1/\sqrt{6}$

7. A variable plane is parallel to the plane $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ and meets the axes in A, B, C. Prove that the circle ABC lies on the cone $\left(\dfrac{b}{c}+\dfrac{c}{b}\right)yz+\left(\dfrac{c}{a}+\dfrac{a}{c}\right)zx+\left(\dfrac{a}{b}+\dfrac{b}{a}\right)xy=0$

8. Find the equation of the right circular cylinder of radius 4 and axis the line $x = 2y = -z$

9. Show that the solutions of $x^2y'''-3xy''+3y'=0,(x>0)$ are linearly independent.

OR

Solve the equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-4)y=0$ in series form.

10. Prove that $4J_n''(x)=J_{n-2}(x)-2J_n(x)+J_{n+2}(x)$ where the symbols have their usual meanings.

11. Apply the power series method to the following differential equation $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

OR

Find the general solution of Legendre's differential equation.

12. Show that $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})=\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{c}$ and deduce $\left[\vec{b}\times\vec{c}\;\; \vec{c}\times\vec{a}\;\; \vec{a}\times\vec{b}\right]=\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$

13. Prove that the necessary and sufficient condition for the function $\vec{a}$ of scalar variable to have a constant direction is $\vec{a}\times\frac{d\vec{a}}{dt}=0$

14. Find the angle between the surface $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2,-1,2)

15. Test the convergence of the series $\sum\frac{(n+1)^n x^n}{n^{n+1}}$

16. Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-1)^n(x-1)^2}{\sqrt{n}}$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2068 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *The figures in the margin indicate __Full Marks__.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem for homogeneous function of two variables. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\text{Cot } u$. [1+4]
2. Find the minimum value of $x^2 + xy + y^2 + 3z^2$ under the condition $x + 2y + 4z = 60$. [5]
3. Change the order of integration and hence evaluate the same.
   $\int_0^a \int_0^x \frac{\cos y\, dydx}{\sqrt{(a-x)(a-y)}}$ [5]
4. Find by double integration, the volume bounded by the plane $z = 0$, surface $z = x^2 + y^2 + 2$ and the cylinder $x^2 + y^2 = 4$. [5]
5. Prove that the plane through the point (α, β, γ) and the line $x = py + q = rz + s$ is given by:
   $\begin{vmatrix} x & py+q & rz+s \\ \alpha & p\beta+q & r\gamma+s \\ 1 & 1 & 1 \end{vmatrix} = 0.$ [5]
6. Find the magnitude and equation of the shortest distance between the lines: [5]
   $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$
7. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 - 3x + 4y - 2z - 5 = 0$, $5x - 2y + 4z + 7 = 0$ as a great circle. [5]

   OR

   Find the equation which touches the sphere $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ at $(1, 2, -2)$ and passes through the point $(1, -1, 0)$. [5]
8. Find the equation of the cone with vertex (α, β, γ) and base $y^2 = 4ax$, $z = 0$ [5]
9. Solve the initial value problem
   $y'' - 4y' + 3y = 10e^{-2x}$, $y(0) = 1$, $y'(0) = 3$. [5]
10. Solve by power series method the differential equation $y'' - 4xy' + (4x^2 - 2)\, y = 0$. [5]

11. Express $f(x) = x^3 - 5x^2 + 6x + 1$ in terms of Legendre's polynomials. [5]

OR

Prove that $\frac{d}{dx}\left[x^{-n}J_n(x)\right] = -x^{-n}J_{n+1}(x)$. [5]

12. Find a set of vectors reciprocal to the following vectors: [5]

$-\vec{i}+\vec{j}+\vec{k},\ \vec{i}-\vec{j}+\vec{k},\ \vec{i}+\vec{j}-\vec{k}$

13. Prove that $\vec{b}\times\vec{c},\ \vec{c}\times\vec{a}$ and $\vec{a}\times\vec{b}$ are coplanar or non-coplanar according as $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar or non-coplanar.

14. Prove that curl $(\vec{a}\times\vec{b}) = \vec{a}$ div $\vec{b} - (\vec{a}.\nabla)\vec{b}$ [5]

OR

If $u = x + y + z$, $v = x^2 + y^2 + z^2$ and $w = xy + yz + zx$, show that [gradu gradv gradew] = 0

15. Test the convergence of the series: [5]

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \dots\dots\dots\dots + \frac{(n+1)}{n^3}x^{n} + \dots\dots\dots$$

16. Find the radius of convergence and the interval of convergence of the power series: [5]

$$\sum_{n=0}^{\infty}\frac{(-1)^n(x-3)^n}{n+1}$$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2067 Mangsir

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's Theorem for a homogeneous function of two independent variables and verify it for the function: [1+4]

$$u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}.$$

2. Find the extreme value of $\phi = x^2+y^2+z^2$ connected by the relation $ax + by + cz = p$ [5]

3. Evaluate: $\iint_R xy\,dx\,dy$ where R is the region over the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant. [5]

4. Transform to polar coordinates and complete the integral $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} (x^2 + y^2)\,dy\,dx$. [5]

OR

Evaluate: $\iiint x^{\ell-1}.y^{m-1}.z^{n-1}dxdydz$

where x, y, z are all positive but $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \le 1$.

5. Find the length of perpendicular from the point (3, –1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also obtain the equation of the perpendicular. [5]

6. Find the length and equation of the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$; $2x - 3y + 27 = 0 = 2y - z + 20$. [5]

7. Find the centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$ is cut by the plane $x - 2y + 2z = 3$. [5]

8. Plane through OX and OY include an angle α. Show that their line of intersection lies on the cone $z^2(x^2 + y^2 + z^2) = x^2y^2 \tan^2 \alpha$. [5]

OR

Find the equation of the right circular cylinder whose guiding curve is the circle $x^2 + y^2 + z^2 - x - y - z = 0, x + y + z = 1$.

9. Solve in series: [5]

$$(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$$

10. Show that: [5]

$$J_{\frac{5}{2}}(x)=\sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x-\frac{3}{x}\cos x\right)$$

11. Show that: [5]

$$P_n(x)=\frac{1}{2^n n}\frac{d^n}{dx^n}\left(x^2-1\right)^n$$

12. Prove that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})+(\vec{a}\times\vec{c})\times(\vec{d}\times\vec{b})+(\vec{a}\times\vec{d})\times(\vec{b}\times\vec{c})=-2\times[\vec{b}\ \vec{c}\ \vec{d}\,]\,\vec{a}$ [5]

13. Prove that the necessary and sufficient condition for the vector function $\vec{a}$ of scalar variable λ to have a constant magnitude is $\left(\vec{a}\,\frac{d\vec{a}}{dt}\right)=0$. [5]

14. Apply the power series method to solve following differential equation [5]

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$$

15. Test the convergence of the series $\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+\ldots\ldots\ldots$ [5]

16. Show that $J_4(x)=\left(\frac{48}{x^3}-\frac{3}{x}\right)J_1(x)+\left(1-\frac{24}{x^2}\right)J_0(x)$. [5]

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03 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2067 Chaitra

Exam.	New Back (2066 Batch Only)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem of homogeneous equation of two variables. If $u = \sin^{-1}\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$. Show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0.$ [1+4]

2. Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 1$. [5]

3. Evaluate $\iint_R xy\,dx\,dy$ where R is the region over the area of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ in the first quadrant. [5]

4. Evaluate the integral by changing to polar co-ordinates. $\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2 + y^2)\, dy\, dx$.

   OR

   Find by triple integral, the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. [5]

5. Prove that $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}) = [\vec{a}\ \vec{b}\ \vec{c}]\,\vec{c}$ and deduce that $[\vec{b}\times\vec{c}, \vec{c}\times\vec{a}, \vec{a}\times\vec{b}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$. [5]

6. Prove that the necessary and sufficient condition for the vector function of a scalar variable t have constant magnitude is $\vec{a}\dfrac{d\vec{a}}{dt} = 0$. [5]

7. The position vector of a moving particle at any point is given by $\vec{r} = (t^2 + 1)\,\vec{i} + (4t - 3)\,\vec{j} + (2t^2 - 6)\,\vec{k}$. Find the velocity and acceleration at $t = 1$. Also obtain the magnitudes. [5]

8. Prove that the lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular if $aa' + cc' + 1 = 0$. [5]

9. Prove that the lines $\dfrac{x-1}{2} = \dfrac{y+1}{-3} = \dfrac{z+10}{8}$ and $\dfrac{x-4}{1} = \dfrac{y+3}{-4} = \dfrac{z+1}{7}$ intersect. Find also their point of intersection and plane through them. [5]

10. Find the centre and radius of the circle $x^2 + y^2 + z^2 + x + y + z = 4$, $x + y + z = 0$. [5]

11. Show that the equation of a cone whose vertex is (α, β, γ) and base the parabola $z^2 = 4ax, y = 0$ is $(\beta z - \gamma y)^2 = 4a(\beta - y)(\beta x - \alpha y)$. [5]

OR

Find the equation of the right circular cylinder of radius 4 and axes of the line $x = 2y = -z$.

12. Test the convergence of the series $\frac{2}{1^p} + \frac{3}{2^p} + \frac{4}{3^p} + \frac{5}{4^p} + \frac{6}{5^p} + \ldots\ldots$ [5]

13. Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{(n+1)}$. [5]

14. Solve $(x+a)^2 \frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$. [5]

15. Solve the initial value problem
$y'' + y' - 2y = -6\sin 2x - 18\cos 2x = 0,\ y(0) = 0,\ y'(0) = 0.$ [5]

16. Show that $J_{-n}(x) = (-1)^n J_n(x)$.

OR

Find the general solution of Legendre's differential equation. [5]

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