TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2079 Ashwin

Exam.	Back		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT, BEI, BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH 551)*

- Candidates are required to give their answers in their own words as far as practicable.
- *Attempt* ***All*** *questions.*
- *The figures in the margin indicate* ***Full Marks.***
- *Assume suitable data if necessary.*

1. Show that $u(x, y) = \sin x \cosh y$ is a harmonic function. Also find its harmonic conjugate $v(x, y)$ such that $u + iv$ is analytic. [5]
2. Define Bilinear transformation. Find the Bilinear transformation that maps $z_1 = -2$, $z_2 = 0$, $z_3 = 2$ into the points $\omega_1 = 0$, $\omega_2 = i$ and $\omega_3 = -i$ respectively. [1+4]
3. State and prove Cauchy's integral theorem. [5]
4. State Taylor's theorem for function $f(z)$ of complex variable z. When does Taylor's series reduce to Maclaurin's series? Find Maclaurin's series expansion of the function $f(z) = \tan z$ upto four terms. [1+2+2]
5. Evaluate $\oint_C \frac{2z-1}{z(z+1)(z-3)}\, dz$, where C is the circle $|z| = 2$ by residue method. [5]
6. Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ by contour integration in the complex plane. [5]
7. Define Z-Transform. Find the Z-Transform of $e^{-bt} \sin \omega t$. [5]
8. State and prove final value theorem for Z-Transform. [5]
9. Find the inverse z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$ by using partial fraction method. [5]
10. Solve the difference equation $x(k+2) - 3x(k+1) + 2x(k) = 4^k$ given that $x(0) = 0$, $x(1) = 1$. [5]
11. A tightly stretched string with fixed ends, $x = 0$ and $x = l$ is initially in position given by $u(x, 0) = u_0 \sin\left(\frac{3\pi x}{l}\right)$. If it is released from rest in this position, find the displacement at any time t at distance x from one end. [10]
12. Derive one dimensional heat equation and find its possible solutions. [10]
13. Show that the Fourier Cosine integral representation of $f(x)=e^{-x}$ is $\int_0^\infty \frac{\cos \omega x}{1+\omega^2}\, d\omega = \frac{\pi}{2} e^{-x}$. [5]
14. Find the Fourier sine transform of e^{-x}, $x \geq 0$ and hence by Parseval's identity, show that $\int_0^\infty \frac{x^2}{(1+x^2)^2}\, dx = \frac{\pi}{4}$. [5]

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INSTITUTE OF ENGINEERING
Examination Control Division
2079 Jestha

Exam.		Back	
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT, BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH 551)*

- Candidates are required to give their answers in their own words as far as practicable.
- *Attempt **All** questions.*
- *The figures in the margin indicate **Full Marks**.*
- *Assume suitable data if necessary.*

1. a) State and prove Cauchy-Reimann equations in cartesian forms. [5]

   b) Show that $u = \sin x\cosh y + 2\cos x\sinh y + x^2 - y^2 + 4xy$ is harmonic and find corresponding analytic function. [1+4]

2. a) Find the linear transformation which maps the points $z_1 = 0, z_2 = -1, z_3 = \infty$ into the points $w_1 = 1, w_2 = i, w_3 = -1$. [5]

   b) State Cauchy's integral formula. Use it to evaluate: [1+4]

   $$\int_C \frac{e^z}{(z-1)(z-3)}dz \text{ where c: } |z|<2.$$

3. a) State Taylor's theorem for complex variable. Expand the Laurent's series of the function $f(z) = \frac{1}{z^2-3z+2}$ in the region $1 < |z| < 2$. [1+4]

   b) By using Cauchy Residue theorem evaluate $\int_C \tan z\, dz$ where c is circle $|z| = 2$. [5]

4. a) State and prove final value theorem for z-transform. [1+4]

   b) Obtain z-transform of sin wt and hence evaluate z-transform of ae^{-at} sin wt. [5]

5. a) Obtain the inverse z-transform of $X(z) = \frac{2z}{(z+1)(z^2+1)}$ by using partial fraction method. [5]

   b) Solve the difference equation: $x(k + 2) + 2x(k + 1) + 3x(k) = 0$ given that $x(0) = 0$ and $x(1) = 2$. [5]

6. Derive one dimensional wave equation and solve it completely. [10]

7. Solve one dimensional heat equation: $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial^2 x}$ under boundary condition $\frac{\partial u}{\partial x} = 0$ when $x = 0$ and $x = l$ and the initial condition $u(x, 0) = x$ for $0 < x < l$. [10]

8. a) Find the Fourier sine transform of $e^{-x}, x \geq 0$ and show that $\int_0^\infty \frac{x\sin mx}{1+x^2}dx = \frac{\pi e^{-m}}{2}$ where $m > 0$. [3+2]

   b) Solve the integral equation: $\int_{-\infty}^{\infty} y(u)\, y(x-u)\, du = e^{-x^2}$. [5]

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2078 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BEL,BEX,BCT BEI,BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH 551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *The figures in the margin indicate __Full Marks__.*
- ✓ *Assume suitable data if necessary.*

1. Define an analytic function f(z) of complex variable z at a point. If $f(z) = u(x,y) + i\,v(x,y)$ is analytic, show that $u_x = v_y$ and $u_y = -v_x$. [1+4]
2. Define conformal mapping. Find the linear transformation which maps the points $z = 0, 1, \infty$ in to the points $w = -3, -1, 1$ respectively. [5]
3. State and proof Cauchy's Integral theorem. [5]
4. Obtain the Taylor's series expansion of the complex function $f(z) = \dfrac{z+1}{(z-3(z-4)}$ about the center $z = 2$ up to four term. [5]
5. State Cauchy residue theorem. Apply it to evaluate $\int_c \dfrac{4-3z}{z(z-1)(z-2)}\,dz$ where C is the circle $|z| = \dfrac{3}{2}$. [1+4]
6. Evaluate integrals $\int_0^\pi \dfrac{1}{3+2\cos\theta}\,d\theta$ by contour integration. [5]
7. If $x(t) = 0$ for $t < 0$, $Z[x(t)] = X(z)$ for $t \geq 0$, then prove that $Z[e^{-at}\,x(t)] = X(ze^{aT})$. [5]
8. Obtain the Z- transform of (i) te^{-at} (ii) sin at [2.5+2.5]
9. Obtain the inverse Z- transform of $X(z) = \dfrac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$ where T is the sampling period. [5]
10. Solve the difference equation $y_{n+2} - 4\,y_{n+1} + 4y_n = 0$ with given condition $y_0 = 0$, $y_1 = 1$. [5]
11. A tightly stretched string with fixed ends $x = 0$ and $x = \ell$ is initially in position given by $u(x,0) = u_0 \sin^3 \dfrac{\pi x}{\ell}$. If it is released from rest in this position, find the displacement at any time t at any distance x from one end. [10]
12. Derive one dimensional heat equation and solve it completely. [10]
13. Obtain the fourier sine and cosine integral of $f(x) = x$ for $0 < x < a$,
    $= 0$ for $x > a$. [5]
14. Find the fourier cosine transform of $f(x) = e^{-x}$, $x > 0$ and hence parseval's identity, show that $\int_0^\infty \dfrac{1}{(1+x^2)^2}\,dx = \dfrac{\pi}{4}$. [5]

Exam.		**Back**	
Level	BE	**Full Marks**	80
Programme	BEL, BEI, BEX, BCT, BGE	**Pass Marks**	32
Year / Part	II / II	**Time**	3 hrs.

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2078 Poush

Subject: - Applied Mathematics *(SH 551)*

- Candidates are required to give their answers in their own words as far as practicable.
- *Attempt All questions.*
- *The figures in the margin indicate Full Marks.*
- *Assume suitable data if necessary.*

Define Harmonic function. Show that the function $u(x, y) = e^x \sin y$ is harmonic and hence construct an analytic function $f = u(x, y) + iv(x, y)$. [5]

Find the linear fractional transformation that maps $z = 2, i, -2$ into the points $w = 1, i, -4$. Also find the fixed points of the transformation. [5]

State and prove Cauchy Integral theorem. [5]

State Laurent's theorem and expand the function $f(z) = \dfrac{z^2+1}{(z-1)(z-2)}$ as a Laurent series in the region $1 < |z| < 3$. [5]

Using Cauchy's residue theorem, evaluate the integral $\int_C \tan z \, dz$ where c is the circle $|z| = 2$. [5]

Using contour integration, evaluate the integral $\int_0^{2\pi} \dfrac{d\theta}{3 + 2\sin\theta}$ [5]

State and prove final value theorem for z-transform. [5]

Find the z-transform of the following sequences for $k \geq 0$: [5]

(i) $k\ a^k$ (ii) $\sin k\theta$

Using the partial fraction decomposition method, find the inverse z-transform of the

$X(z) = \dfrac{3z^3 + z}{(z-1)^2(z-2)}$ [5]

Using the z-transform technique, solve the following difference equation:

$y(k+2) - 4y(k+1) + 3y(k) - 2^k = 0$, with $y(0) = 0$, $y(1) = 1$. [5]

Derive one dimensional wave equation and solve it completely. [10]

A rod of length L has its end at A and B maintained at 0°C and 100°C respectively until steady state prevails. If B is suddenly reduced to 0°C, then find the temperature at a distance x from the end A at time t. [10]

Find the Fourier sine transform of the function $f(x) = e^{-mx}$ $(x > 0, m > 0)$ and hence show that $\int_0^\infty \dfrac{\alpha \sin(\alpha x)}{\alpha^2 + \beta^2} d\alpha = \dfrac{\pi}{2} e^{-mx}$ [5]

Starting from the Fourier cosine transform of $f(x) = e^{-x}$ for $x > 0$, show that

$\int \dfrac{1}{\ldots} dx = \dfrac{\pi}{\ldots}$ [5]

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2078 Baishakh

Exam.	Back		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT, BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics (SH 551)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *The figures in the margin indicate Full Marks.*
- ✓ *Assume suitable data if necessary.*

1. a) Show that the function $u = e^x \cos y$ is harmonic, find its harmonic conjugate and hence construct the corresponding analytic function. [1+3+1]

   b) Find the linear transformation which maps the points $z_1=2$, $z_2=i$, $z_3=-2$ into the points $w_1=1$, $w_2=i$, $w_3=-1$. [5]

2. a) State Cauchy's Integral formula. Use it to evaluate: $\int_c \frac{e^z}{(z-1)(z-3)}dz$ where $c:|z| = 2$. [1+4]

   b) Expand $f(z) = \cos z$ in Talyor's series about $z = \frac{\pi}{2}$. [5]

3. a) State Cauchy's Residue theorem. Use it to evaluate: $\int_c \tan z\,dz$ where c is circle $|z|=2$. [5]

   b) Evaluate by using contour integration in a complex plane: $\oint_0^{2\pi} \frac{2d\theta}{2+\cos\theta}$ [5]

4. a) Find the z-transform of: [2×2.5]

   (i) $te^{-\alpha t}, t \geq 0$

   (ii) $\sin\alpha t$

   b) State initial value theorem for z-transform. Find the initial value x(0) and x(1) for the function: $X(z) = \frac{(1-e^{-T})z^{-1}}{(1-z^{-1})(1-e^{-T}z^{-1})}$. [1+4]

5. a) Find the inverse z-transform of $\frac{z}{(z+1)^2(z-1)}$. [5]

   b) Solve the difference equation $x(k+2)-x(k+1)+0.25x(k)=u(k)$ given that $x(0)=1$ and $x(1)=2$ and u(k) is unit step function. [5]

6. Derive one dimensional wave equation and solve it completely. [5+5]

7. A rectangular plate with insulated surfaces is 10cm wide and so long compared to its width introducing an appreciable error. If the temperature along the short edge y = 0 is given by $U(x,0)=\begin{pmatrix} 20x, & 0<x\leq 5 \\ 20(10-x), & 5<x<10 \end{pmatrix}$ while the two long edges x=0 and x=10 as well as the other, short edges are kept at 0°C. Find the steady state temperature at any point (x,y) of the plate. [10]

8. a) Obtain the Fourier integral of $f(x)=\begin{cases} \cos x, & for\ |x|<\frac{\pi}{2} \\ 0, & for\ |x|>\frac{\pi}{2} \end{cases}$. [5]

   b) Show that the Fourier Cosine Integral representation of $f(x)=e^{-x}$ is

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2077 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT, BEI, BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics (SH 551)

dates are required to give their answers in their own words as far as practicable.
pt All questions.
figures in the margin indicate ***Full Marks.***
me suitable data if necessary.

State Cauchy-Riemann equation in polar form. Prove that $f(z) = |z|$ is not an analytic function. [1+4]

Find the linear transformation which maps the points $z = 0, 1, \infty$ into the points $w = -3, -1, 1$ respectively. Find also fixed point of the transformation. [4+1]

State and prove Cauchy's Integral Formula. [1+4]

Find the Laurent's series of $f(x) = \dfrac{z^2 - 1}{(z+2)(z+3)}$ in the region $2 < |z| < 3$. [5]

State Cauchy Residue theorem and hence evaluate the integral $\displaystyle\int_C \frac{z-1}{(z+1)^2(z-2)}dz$ where $C : |z-i| = 2$. [1+4]

Using counter Integration, evaluate $\displaystyle\int_0^{2\pi} \frac{1}{2+\cos\theta}d\theta$ in the complex plane. [5]

State and prove initial value theorem of z-transform. [5]

Find the z-transform of the following sequence for $t \geq 0$. [2.5+2.5]
(i) te^{-at} (ii) sinat

Find the inverse z-transform of the function $\dfrac{2z^3 + z}{(1-2)^2(z-1)}$. [5]

Solve the difference equation $x(k+2) - 4x(k+1) + 4x(k) = 0$ with conditions $x(0) = 1$, $x(1) = 0$. [5]

Derive one dimensional heat equation and solve it completely. [5+5]

A string is stretched and fastened to two points apart. Motion is started by displacing the string in the form $u(n, 0) = u_0 \sin\dfrac{\pi x}{l}$ from which it is released at time t=0. Show that the displacement at any point at a distance x from one end at a time t is given by $u(x,t) = u_0 \sin\dfrac{\pi x}{l}\cos\dfrac{\pi c t}{l}$. [10]

a) Define the complex form of Fourier integral of a given function with usual notation. Find the Fourier integral representation of the function $f(x) = \begin{pmatrix} 1 & if & |x| < 1 \\ 0 & if & |x| > 1 \end{pmatrix}$ and hence evaluate $\displaystyle\int_0^{\infty} \frac{\sin w}{w} dw$. [1+3+1]

b) Find the Fourier sine transformation of $f(x)=e^{-|x|}$ and hence evaluate the integral

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2076 Baisakh

Exam.	Back		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT, BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH 551)*

ndidates are required to give their answers in their own words as far as practicable.
pt All questions.
figures in the margin indicate Full Marks.
me suitable data if necessary.

ain polar form Cauchy-Riemann equations for function of complex variable. [5]

the linear fractional transformation which maps the points z=0,1,∞ into the points -3,-1,1 respectively. [5]

e Complex integration. How does it differ from real integration? Derive Cauchy gral formula for function f(z). [1+1+3]

e Laurent's Series for the function of complex variable. Obtain Taylor's series for ction [1+4]

$$f(z)=\frac{z}{z^2+4} \text{ about } z=i$$

e Cauchy residue theorem. Apply it to evaluate $\oint_c \tan z\,dz$, where c is the region |z|=2. [1+4]

uate integral $\int_0^{2\pi}\frac{d\theta}{a+\sin\theta}$; a>1 by contour integration in complex plane. [5]

in z-transform of sin ωt and hence obtain z-transform of $e^{at}\sin\omega t$. [3+2]

in the inverse z-transform of $X(z)=\frac{z^2}{(z-1)^2(z-e^{-aT})}$. [5]

e and prove shifting to the right theorem for z-transform. [5]

e the difference equation: [5]

$x(k+2)-x(k+1)+0.25x(k)=u(k)$ where x(0)=1 and x(1)=2 and u(k) is a unit step ction; by z-transform method.

Fourier integral of the function [5]

$$f(x)=\begin{cases}0 & \text{if } x<0\\ e^{-x} & \text{if } x<0\end{cases}$$

the Fourier Sine transform of e^{-x}, x≥0 and hence show that $\int_0^\infty \frac{x^2}{(1+x^2)^2}dx=\frac{\pi}{4}$ [5]

e the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ with the given boundary conditions; u(0,t)=0, t)=0, u(x,0)=0 and $\left(\frac{\partial u}{\partial t}\right)_{t=0}=3(Lx-x^2)$. [10]

dimensional heat equation and solve it completely. [10]

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Examination Control Division

2075 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BEL,BEX, BCT, BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Define harmonic function of complex variable. Show that $u(x,y) = y^3 - 3x^2y$ is harmonic and find corresponding analytic function. [1+4]
2. Define conformal mapping for function of complex variable. Show that function of complex variable $w = iz$ is transformed through an angle $\frac{\pi}{2}$ in w-plane. [1+4]
3. State and prove Cauchy's integral theorem. [5]
4. Define Laurent's Series for the function of complex variable. Find Laurent's series of the function $f(z) = \dfrac{z}{(z+2)(z+3)}$ in the region $2 < |z| < 3$. [1+4]
5. Define pole of order m for function of complex variable. Find residues of $f(z) = \dfrac{z^2 - 2z}{(z+1)^2(z^2+1)}$ at its poles. [1+4]
6. Evaluate $\int_{-\infty}^{\infty} \dfrac{x^2}{(x^2+1)(x^2+4)}dx$ by contour integration in the complex plane. [5]
7. Find the Z-transform of: [3+2]

   i) t^2e^{at}

   ii) $e^{-at}\cos wt$
8. Find the inverse Z-transform of: [2.5+2.5]

   i) $X(z) = \dfrac{2z^2 - 5z}{(z-2)(z-3)}$ (By partial fraction method)

   ii) $X(z) = \dfrac{z^{-2}}{(1-z^{-1})^3}$ (By inversion integral method)
9. State final value theorem for Z-transform. Obtain Z-transform of $(1-e^{-at})$; a>0 and hence evaluate $x(\infty)$ by using final value theorem. [1+4]
10. Solve the difference equation: [5]

    $x(k+2) - 3x(k+1) + 2x(k) = 0$; given that $x(0) = 0$ and $x(1) = 1$ by using z-transform method.
11. Find the Fourier integral of the function: [5]

    $$f(x) = \begin{cases} 1, & \text{for } 0 < x < \pi \\ 0, & \text{for } x > \pi \end{cases}$$

12. Find the Fourier transform of e^{-x^2}. Also verify the convolution theorem for $f(x)=e^{-x^2}$ and $g(x)=e^{-x^2}$ [5]

13. Derive one dimensional wave equation and solve it completely. [10]

14. Solve completely the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ under the conditions: [10]

$$u(0,y)=u(l,y)=u(x,o)=0, u(x,\infty)=\sin\left(\frac{n\pi x}{l}\right)$$

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14 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2075 Baisakh

Exam.	Back		
Level	BE	Full Marks	80
Programme	BGE, BEL, BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) Define harmonic function. Is V = arg(z) is harmonic? If yes, find a corresponding harmonic conjugate. [1+1+3]

   b) Define conformal mapping. Find the bilinear transformation which maps the points z = 0, 1, ∞ into the points w = -3, -1, 1 respectively. [1+4]

2. a) Distinguish between Cauchy integral Theorem and Cauchy integral formula. Using Cauchy integral formula evaluate $\int_c \frac{e^z}{(z+1)(z-2)}dz$ where C is the circle $|z-1|=3$. [1+4]

   b) State and Prove Taylor's series for function of complex variable. [5]

3. a) Define an isolated pole. Using Cauchy's residue theorem evaluate $\int_c \frac{z-1}{(z+1)^2(z-2)}dz$ where C is the circle $|z-i|=2$. [5]

   b) Evaluate the integral by contour integration: [5]

   $$\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)(x^2+4)}dx$$

4. a) Obtain the z-transform of (1-e^{-at}), a > 0 and hence evaluate x(∞) by using final value theorem. [2+3]

   b) Obtain the inverse z-transform of:

   $X(z)=\frac{2z^3+z}{(z-2)^2(z-1)}$ by using partial fraction method. [5]

5. a) Define z-transform of function f(t). Find the z-transform of following sequences: [1+2+2]

   (i) $f(k)=\left\{\begin{matrix}15,10,7,4,1,-1,3,6\\ \uparrow\end{matrix}\right\}$

   (ii) $f(k)=\begin{cases}5^k & ; \quad k<0\\ 2^k & ; \quad k\geq 0\end{cases}$

   b) Solve the difference equation by the application of z-transform:

   $x(k+2)+3x(k+1)+2x(k)=0$ with conditions x(0) = 0, x(1) =1. [5]

6. a) A tightly stretched string with fixed ends at x = 0 and x = l is initially at rest in its equilibrium position. Find the deflection u(x, t) if it is set vibrating by giving to each of its points a velocity $3(lx-x^2)$. [10]

   b) Derive two dimensional heat equation. [10]

7. a) Obtain the Fourier sine integral representation of $e^{-x}\cos x$ and hence show that $\int_0^{\infty}\frac{\omega^3 \sin\omega x}{\omega^4+4}d\omega=\frac{\pi}{2}e^{-x}\cos x, \quad x>0.$ [5]

   b) Find the Fourier Cosine transform of $f(x) = e^{-x}$, $x > 0$ and hence by Parseval's identity, show that [5]

$$\int_0^{\infty}\frac{1}{\left(1+x^2\right)^2}dx=\frac{\pi}{4}.$$

14 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2074 Bhadra

Exam.	**Regular**		
Level	BE	**Full Marks**	80
Programme	BGE, BEL, BEX, BCT	**Pass Marks**	32
Year / Part	II / II	**Time**	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) Define an analytic function for a function of complex variable. Derive Cauchy Riemann equations in Cartesian form. [1+4]

   b) Define linear fractional mapping. Find bilinear mapping which maps the points z = 0, 1, -1 to w = i, 2, 4. [1+4]

2. a) State and Prove Cauchy integral theorem. [5]

   b) Point out difference between Taylor's series and Laurent's series. Find Laurent' series of function $f(z)=\frac{\sin z}{z^6}$, $0<|z|<TR$ [1+4]

3. a) Define pole of order m. Using Cauchy's residue theorem evaluate $\int_c \cot z\ dz$; where C is $|z|=1$. [1+4]

   b) Using Counter integration evaluate,

   $$\int_{-\infty}^{\infty}\frac{dx}{(1+x^2)^2}.$$ [5]

4. a) Find the z-transform of:

   (i) cosat (ii) te^{-at} [2+3]

   b) State final value theorem. If x(t) = 0 for t < 0 and Z[x(t)] = X(z) for t ≥ 0 then prove that:

   $$Z[x(t+nT)]=z^n\left[X(z)-\sum_{k=0}^{n-1}x(kT)z^k\right].$$ [1+4]

5. a) Obtain inverse Z-transform of $\frac{z(3z^2-6z+4)}{(z-1)^2(z-2)}$. [5]

   b) Solve the difference equation by the application of z-transform:

   x (k+2) -4x (k+1) + 4x(k) = 0; with conditions x(0) = 1; x(1) = 0. [5]

6. a) Derive one dimensional wave equation and solve it completely. [5+5]

   b) A uniform rod of length ℓ has its end maintained at a temperature 0°C and the initial temperature of the rod is:

   $$u(x,0)=3\sin\frac{\pi x}{\ell} \quad \text{for } 0<x<\ell.$$

   Find the temperature u(x, t). [10]

7. a) Find Fourier integral of the function

   $$f(x)=\begin{cases}1 & \text{if } |x|<1\\ 0 & \text{if } |x|>1\end{cases}$$ [5]

   b) Verify the convolution theorem for Fourier transform for the functions

TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2073 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT, BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject*: - Applied Mathematics *(SH551)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) Define harmonic function of complex variable. Determine the analytical function

   $f(z) = u + iv$ if $u = y^3 - 3x^2y$ [1+4]

   b) Derive Cauchy-Reimann equations if function of complex variable $f(z) = u + iv$ is analytic in cartesian form. [5]

2. a) What do you mean by conformal mapping? Find the linear transformation which maps points $z_1 = 1, z_2 = i, z_3 = -1$ into the points $w_1 = 0, w_2 = 1, w_3 = \infty$. [1+4]

   b) State and prove Cauchy's integral formula. [5]

3. a) State Taylor's theorem. Find the Laurent's series representation of the function

   $f(z) = \dfrac{z}{(z+1)(z+2)}$ in the annular region between $|z| = 1$ and $|z| = 2$. [1+4]

   b) Define zero of order m of function of complex variable .Determine the poles and residue at poles of the functions $f(z) = \dfrac{1+z}{(z+2)(1-z)^2}$. [1+4]

   OR

   Evaluate the real integral $\int_{-\infty}^{\infty} \dfrac{x^2}{(1+x^2)^3} dx$ by contour integration in the complex plane. [5]

4. a) Define z-transform. How does it differ from Fourier transform? Obtain z-transform of

   (i) $t^2 a^t$ (ii) $\cos at$ [1+1+1.5+1.5]

   b) State initial value theorem for z transform. Find the initial value x(0) and x(1) for the function. [1+4]

   $$X(z) = \frac{(1-e^{T})z^{-1}}{(1-z^{-1})(1-e^{-T}z^{-1})}$$

5. a) Obtain the inverse z-transform of $X(z) = \dfrac{3z^3 + 2z}{(z-3)^2(z-2)}$ by using inversion integral method. [5]

   b) Apply method of z-transform to solve the difference equation [5]

   $x(k+2) - 4x(k+1) + 4x(k) = 0; x(0) = 0, x(1) = 1$

6. Solve completely one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions: [10]

$u(0,t) = 0, u(l,t) = 0, u(x,0) = 0$ and $\left(\frac{\partial u}{\partial t}\right)_{\text{at } t=0} = 3(lx - x^2)$

7. Derive one dimensional heat equation and solve it completely. [10]

8. a) State convolution theorem for Fourier transform. Give its importance with suitable example. [2+3]

   b) Find the Fourier cosine integral of the function $f(x) = e^{-kx} (x > 0, k > 0)$ and hence show that $\int_0^\infty \frac{\cos\omega x d\omega}{k^2 + \omega^2} = \frac{\pi}{2k} e^{-kx}; x > 0, k > 0$ [5]

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2072 Ashwin

Exam.	Regular		
Level	BE	**Full Marks**	**80**
Programme	BEL, BEX, BCT, BGE	**Pass Marks**	32
Year / Part	II / II	**Time**	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) If $u = (x-1)^3 - 3xy^2 + 3y^2$, determine V so that u + iv is an analytic function of x+iy. [5]

   b) Define an analytic function. Express Cauchy Riemann equations $u_x = v_y$ and $u_y = -v_x$ in polar from. [5]

2. a) Find the bilinear transformation which maps points $z_1 = 1, z_2 = i, z_3 = -1$ into the points $w_1 = i, w_2 = -1, w_3 = -i$ respectively. [5]

   b) Evaluate $\int_0^{1+i} (x^2 + iy)dz$ along the path $y = x^2$ [5]

3. a) Express $f(z) = \dfrac{1}{(z^2 - 3z + 2)}$ as Laurent's series in the region $1 < |z| < 2$. [5]

   b) Evalute $\int_0^{2\pi} \dfrac{1}{5 - 4\sin\theta} d\theta$ by contour integration method in complex plane. [5]

4. a) Find z-transform of: [5]
   i) te^{-at}
   ii) sinat

   b) State and prove final value theorem for z- transform. [5]

5. a) Find the inverse z-transform of $\dfrac{2z^2 - 5z}{(z-2)(z-3)}$ by using partial fraction method. [5]

   b) Solve difference equation $x(k+2) - 3x(k+1) + 2x(k) = 4^k$ for x(0) = 0 and x(1) = 1. [5]

6. Derive one dimensional wave equation and obtain its solution. [10]

7. Solve one dimensional heat equation: [10]

   $\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ under the conditions:

   i) u is not infinite as $t \to \infty$
   ii) $\dfrac{\partial u}{\partial x} = 0$ for x = 0 and $x = l$
   iii) $u(x,0) = lx - x^2$ for t = 0; between x = 0 and $x = l$

8. a) Find Fourier integral representation of $f(x) = e^{-x}, x > 0$ and hence evaluate $\int_0^\infty \dfrac{\cos(sx)}{s^2 + 1} ds$ [5]

   b) Find the Fourier cosine transform of $f(x) = e^{-|x|}$ and hence, by Parseval's identity, shown that $\int_0^\infty \dfrac{1}{(1+x^2)^2} dx = \dfrac{\pi}{4}$ [5]

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35 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2071 Bhadra

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT, BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Determine the analytic function $f(z) = u + iv$ if $u = \log\sqrt{x^2+y^2}$.
2. State and prove Cauchy's integral formula.
3. Find the Taylor's series of $f(z) = \frac{1}{1-z}$ about $z = 3i$.
4. Evaluate the integral: $\oint_C \frac{z^2 dz}{(z+1)(z+3)}$ where C: $|z| = 4$, using residue theorem.
5. Define conformal mapping, show that $w = \frac{az+b}{cz+d}$ is invariant to

$$\left(\frac{w-w_1}{w-w_3}\right)\times\left(\frac{w_2-w_3}{w_2-w_1}\right)=\left(\frac{z-z_1}{z-z_3}\right)\times\left(\frac{z_2-z_3}{z_2-z_1}\right)$$

6. Using contour integration, evaluate real integral: $\int_{-\infty}^{+\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$
7. Find the z-transform of $x(z) = \cosh t \sinh t$.
8. State and prove "final value theorem" for the z-transform.
9. Find the inverse z-transform of $x(z) = \frac{z}{z^2+7z+10}$.
10. Using z-transform solve the difference equation:
    $x(K+2) + 6x(K+1) + 9x(K) = 2^K$; $x_0 = x_1 = 0$.
11. Derive one-dimensional heat equation.
12. Solve the wave equation for a tightly stretched string of length 'l' fixed at both ends if the initial deflection in $y(x, 0) = lx - x^2$ and the initial velocity is zero.
13. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{2y^2} = 0$ under the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$, $u(x, a) = \sin\left(\frac{n\pi x}{l}\right)$
14. Derive the wave equation (vibrating of a string).
15. Find the Fourier cosine transform of $f(x) = e^{-|m|x}$ and hence show that $\int_0^\infty \frac{\cos p\gamma}{\gamma^2+\beta^2} d\gamma = \frac{\pi}{2\beta} e^{-p\beta}$.
16. Find the Fourier integral representation of the function $f(x) = e^{-x}$, $x \geq 0$ with $f(-x) = f(x)$.
    Hence evaluate $\int_0^\infty \frac{\cos(sx)}{s^2+1} ds$.

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10/13 M.

12 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2071 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT, BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) Determine the analytic function $f(z) = u + iv$ if $u = 3x^2y - y^3$. [5]

   b) Find the linear transformation which maps the points $z = 0, 1, \infty$ into the points $w = -3, -1, 1$ respectively. Find also fixed points of the transformation. [5]

2. a) State and prove Cauchy's integral formula. [5]

   b) Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$. [5]

3. a) Find the first four terms of the Taylor's series expansion of the complex function $f(z) = \frac{z+1}{(z-3)(z-4)}$ about the centre $z = 2$. [5]

   b) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$. [5]

   OR

   Evaluate $\int_0^{2\pi} \frac{1}{\cos\theta + 2} d\theta$ by contour integration in the complex plane.

4. Derive one dimensional heat equation $u_t = c^2u_{xx}$ and solve it completely. [10]

5. Find all possible solution of Laplace equation $u_{xx} + u_{yy} = 0$. Using this, hence solve $u_{xx} + u_{yy} = 0$, under the conditions $u(0, y) = 0$, $u(x, y) = 0$ when $y \to \infty$ and $u(x, 0) = \sin x$. [10]

6. a) Find the z-transform of $\sin K\theta$. Use it to find the $z[a^K \sin K\theta]$. [5]

   b) If $z[x(K)] = \frac{2z^2 + 3z + 12}{(z-1)^4}$, find the value of $x(2)$ and $x(3)$. [5]

7. a) Find the inverse z-transform of $x(z) = \frac{3z^3 + 2z}{(z-3)^2(z-2)}$ by using inversion integral method. [5]

   b) Using z-transform solve the difference equation $x(K + 2) - 4x(K + 1) + 4x(K) = 2^K$ given that $x(0) = 0$, $x(1) = 1$. [5]

8. a) Find the Fourier sine integral of the function $f(x) = e^{-Kx}$ and hence show that [5]

$$\int_0^{\infty} \frac{\lambda \sin \lambda x}{\lambda^2 + \beta^2} d\lambda = \frac{\pi}{2} e^{-Kx}, \quad x > 0, K > 0$$

   b) Find the Fourier sine transform of e^{-x}, $x \geq 0$ and hence show that [5]

$$\int_0^{\infty} \frac{x \sin mx}{x^2 + 1} dx = \frac{\pi}{2} e^{-m}, \quad m > 0$$

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25 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2070 Bhadra

Exam.	Regular		
Level	BE	**Full Marks**	80
Programme	BEL, BEX, BCT	**Pass Marks**	32
Year / Part	II / II	**Time**	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Show that $u(x, y) = x^2 + 2xy - y^2$ is a harmonic function and determine $v(x, y)$ in such a way that $f(z) = u(x, y) + iv(x, y)$ is analytic. [5]
2. Define complex integral. State and prove Cauchy integral formula. [5]

OR

Obtain bilinear transformation which maps $-i$, o, i to -1, i, 1. [5]

3. Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is $|z| = 3$ using Cauchy's integral formula. [5]
4. Obtain the Laurent series which represents the function $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ $2 < |z| < 3$. [5]
5. Find the Laurent series of $f(z) = \frac{1}{4+z^2}$ about the point $z = i$. [5]
6. State and prove Taylor series of a function $f(z)$. [5]
7. Derive one dimensional wave equation $u_{tt} = c^2 u_{xx}$ and solve it completely. [10]
8. Solve one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary condition $\frac{\partial u}{\partial x} = 0$ when $x = 0$ and $x = L$ and initial condition $u(x, 0) = x$ for $0 < x < L$. [10]
9. Find Z transform of (a) te^{-at} and (b) sin at. [5]
10. Find the inverse z-transform (a) $\frac{z-4}{(z-1)(z-2)^2}$ (b) $\frac{z}{z^2 - 3z + 2}$. [5]
11. Obtain the Z transform of $x(t) = (1 - e^{-at})$, $a > 0$ and hence evaluate $x(\infty)$ by using final value theorem. [5]
12. Solve using z-transform the difference equation $x(K + 2) + 2x(K + 1) + 3x(K) = 0$. [5]
13. Find the Fourier sine transform of $f(x) = e^{-x}$, $x \geq 0$ and hence evaluate $\int_0^\infty \frac{x \sin x}{(1+x^2)} dx$. [5]
14. State and prove convolution theorem of Fourier transform. [5]

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25 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2070 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Define analytic function. Show that the function $f(z) = \frac{1}{z^4}$ is analytic except $z = 0$ [5]

2. Define complex integral. Evaluate $\int_c \log z\, dz; c: |z| = 1$ [5]

OR

Obtain a bilinear transformation which maps -i, 0, i to -1, i, 1.

3. Evaluate $\int_0^{1+i} (x^2 + iy)\, dz$ along the path $y = x$. [5]

4. Find the Taylor series of $f(z) = \frac{1}{4+z^2}$ about the point $z = i$. [5]

5. Evaluate the integrals by residue theorem $\int_c \frac{1-\cos z}{z^3} dz$ [5]

6. State Cauchy's Residue theorem and use it to evaluate $\int_c \frac{z^2}{3+4z+z^2} dz$ where C is $|z|=2$ [5]

OR

Evaluate $\int_0^{2\pi} \frac{d\theta}{\cos\theta + 2}$ by contour integration in complex plane.

7. Derive the one dimensional wave equation. [10]

8. A rod of length L has its ends A and B maintained at 0° and 100°respectively until steady state prevails. If the changes are made by reducing the temperature of end B to 85° and increasing that of end A to 15°, then find the temperature distribution in the rod at a time t. [10]

9. Find the z-transform of (i) $e^{-at} \sin wt$ (ii) cos at [5]

10. Obtain inverse Z-transform of (i) $\frac{z+2}{(z-2)(z-3)}$, (ii) $\frac{z}{(z-2)(z-1)}$ [5]

11. If $x(k) = 0$ for $k < 0$ and $Z\{x(k)\} = X(z)$ for $k > 0$ then prove that $Z\{x(k+n)\} = z^n X(z) - z^n \sum_{k=0}^{n-1} \chi(k) z^{-k}$ where $n = 0, 1, 2....$ [5]

12. Solve the difference equation $x(k+2) - 4x(k+1) + 4x(k) = 0$ with conditions, $x(0) = 0, x(1) = 1$ [5]

13. Find the cosine transform of $f(x) = e^{-mx}$ $m > 0$ show that $\int_0^\infty \frac{\cos pr}{r^2 + B^2} = \frac{\Pi}{2B} e^{-PB}$ [5]

14. Find the Fourier transform of $g(x) = \begin{cases} 1 - x^2 & \text{if } -1 < x < 1; \\ 0, & \text{if otherwise.} \end{cases}$ [5]

and hence use it to evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) \cos(x/2) dx$

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25 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2069 Bhadra

Exam.	**Regular (2066 & Later Batch)**		
Level	BE	**Full Marks**	80
Programme	BEL, BEX, BCT	**Pass Marks**	32
Year / Part	II / II	**Time**	3 hrs.

Subject: - Applied Mathematics ***(SH551)***

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *__All__ questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Determine the analytic function $f(z) = u(x,y) + iv(x,y)$ if $u(x,y) = x^2 - y^2$.

2. Define complex integral. Evaluate: $\oint_C (z+1)dz$ where C is the square with vertices at $z = 0$, $z = 1$, $z = 1+i$ and $z = i$.

OR

Find linear fractional transformation mapping of: $-2 \mapsto \infty, 0 \mapsto \frac{1}{2}, 2 \mapsto \frac{3}{4}$.

3. a) State Cauchy's integral formula and evaluate the integral $\oint_C \frac{4-3z}{z(z-1)(z-2)}dz$, where C is circle $|z| = \frac{3}{2}$.

   b) Obtain the Laurent series which represents the function $f(z) = \frac{1}{(1+z^2)(z+2)}$ when $1 < |z| < 2$.

4. a) Find the Taylor's series expansion of $f(z) = \frac{1}{z^2+4}$ about the point $z = i$.

   b) Evaluate $\int_C \tan z \, dz$ where C is a circle $|z| = 2$ by Cauchy's residue theorem.

OR

Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$ by contour integration in the complex plane.

5. Find the z-transforms of: (i) cos h (at) sin (bt) (ii) n.(n-1); n = k

6. Find the inverse z-transforms of: (i) $\frac{Z}{Z^2 - 3Z + 2}$ (ii) $\frac{Z}{(Z+1)^2(Z-1)}$.

7. a) State and prove convolution theorem for z-transform.

   b) Solve by using z-transform the difference equation $x(k+2) + 2x(k+1) + 3x(k) = 0$, given that $x(0) = 0$ and $x(1) = 2$

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8. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ given that u = 0 as $t \to \infty$ as well as u= 0 at x = 0 and x = l.

9. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which satisfies the condition u(0,y) = u(L,y) = u(x,0) = 0 and $u(x,a) = \sin\left(\frac{n\pi x}{L}\right)$.

OR

The diameter of a semi-circular plate of radius a is kept at 0°C and the temperature at the semi-circular boundary is u_0. Find the steady state temperature in the plate.

10. Find the Fourier integral representation of the function $f(x) = e^{-x}$, $x \geq o$ with $f(-x) = f(x)$.

Hence evaluate $\int_0^\infty \frac{\cos(sx)}{s^2+1} ds$.

11. Find the Fourier transform of:

$f(x) = 1-x^2$, $|x|<1$

$= 0$, $|x|>1$ and hence evaluate

$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2} dx$.

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25 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2068 Bhadra

Exam.	**Regular**		
Level	BE	**Full Marks**	80
Programme	BEL, BEX, BCT	**Pass Marks**	32
Year / Part	II / II	**Time**	3 hrs.

Subject: - Applied Mathematics

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *__All__ questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. a) State necessary conditions for a function f(z) to be analytic. Show that the function $f(z) = \log z$ is analytic everywhere except at the origin.

   b) Find the linear fractional transformation that maps the points $z_1 = -i$, $z_2 = 0$ and $z_3 = i$ into points $w_1 = -1$, $w_2 = i$, $w_3 = 1$ respectively.

2. a) State and prove Cauchy's integral formula.

   b) Write the statement of Cauchy's integral formula. Use it to evaluate the integral $\oint_c \frac{e^z}{(z-1)(z-3)} dz$ where C is the circle $|z| = 2$.

3. a) Write the statement of Taylor's theorem. Find the Laurent series for the function $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region $1 < |z| < 2$.

   b) State Cauchy-residue theorem. Using it evaluate $\oint_c \frac{\sin z}{z^6} dz$ where C: $|z| = 1$.

   OR

   Evaluate $\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta} d\theta$ by contour integration in the complex plane.

4. a) Show that the Z-transform of $\cos k\theta$ is $\frac{z(z - \cos\theta)}{z^2 - 2z\cos\theta + 1}$. Use this result to find Z-transform of $a^k \cos k\theta$.

   b) Obtain the inverse Z-transform of $\frac{2z^3 + z}{(z-2)^2(z-1)}$, using partial fraction method.

5. a) Solve the difference equation $x(k + 2) - x(k + 1) + 0.25x(k) = u(k)$ where $x(0) = 1$ and $x(1) = 2$ and u(k) is unit step function.

   b) State and prove shifting theorem of z-transform.

6. Derive one-dimensional wave equation governing transverse vibration of string and solve it completely.

7. Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions:

a) u is not infinite as $t \to \infty$

b) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = l$ and

c) $u(x,0) = lx - x^2$ for $t = 0$ between $x = 0$ and $x = l$

OR

The diameter of a semi circular plate of radius a is kept at 0°C and temperature at the semi circular boundary is T°C. Show that the steady temperature in the plate is given by $u(r, \theta)\ \frac{4T}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \left(\frac{r}{a}\right)^{2n-1} \sin(2n-1)\theta$

8. a) Find the Fourier cosine integral representation of the function $f(x) = e^{-kx}$ $(x > 0, k > 0)$ and hence show that

$$\int_0^{\infty} \frac{\cos \omega x}{k^2 + \omega^2} d\omega = \frac{\pi}{2k} e^{-kx} \quad (x > 0, K > 0)$$

b) Obtain Fourier sine transform of e^{-x}, $(x > 0)$ and hence evaluate $\int_0^{\infty} \frac{x^2}{(1+x^2)^2} dx$.

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22 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2067 Mangsir

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt any* ***Six*** *questions.*
- ✓ ***All*** *questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. a) State Cauchy - Riemann equations in polar form. Show that f(z) = sinz is analytic in the entire z-plane.

   b) State and prove Cauchy's integral formula.

2. a) State Laurent series. Find Taylor series of f(z) = cosz about $z = \frac{\pi}{4}$.

   b) Define pole of order m. Find the residue of $f(z) = \frac{Z^2 e^z}{(Z-2)^3}$ at its pole.

3. a) Determine the Z-transform of

   i) $t^2 e^{-at}$

   ii) e^{-at} coswt

   b) State initial value theorem for Z-transform. If Z-transform of a function is given by

   $X(z) = \frac{(1-e^{-t})z^{-1}}{(1-z^{-1})(1-e^{-t}z^{-1})}$, determine x(0), x(1) and x(2).

4. a) Find inverse Z- transform of

   i) $x(z) = \frac{z+2}{z^2 - 5z + 6}$ (by partial fraction method)

   ii) $x(z) = \frac{z+2}{z^2 + 7z + 10}$ (by inversion integral method)

   b) Solve the difference equation: x(k+2) –4x(k+1) +4x(k) = 0 Where x(o) = 1 and x(1) = 0.

5. Derive one dimensional wave equation and obtain its solution.

6. Solve: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions, u(o,y) = u(ℓ,y) = u(x,o) = 0, and $u(x,a) = \sin\left(\frac{n\pi x}{\ell}\right)$.

7. Define convolution for Fourier transform. Verify convolution theorem for $f(x) = g(x) = e^{-x^2}$.

8. Maximize: $z = x_1 + 3x_2$ subject to

   $x_1 + 2x_2 \le 10$, $x_1 \le 5$, and $x_2 \le 4$; $x_1, x_2 \ge 0$

   by using simplex method.

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